BOOK-2

MATHEMATICAL ANALYTICS

1. A number consists of two digits with 4 in the units place. If the two digits are interchanged it is $\frac{4}{7}$ times the original number. The number is

   (a) 84 (b) 74 (c) 64 (d) 54

2. In an election A, B and C together got 2000 votes. A and C together got 1500 votes and B and C together got 800 votes. The votes obtained by C are

   (a) 200 (b) 300 (c) 400 (d) 500

3. Expressed as a decimal, 145% is equal to :

   (a) 145 (b) 14.5 (c) 1.45 (d) 0.145

4. Which one of the following is a prime number?

   (a) 823 (b) 343 (c) 221 (d) 143

5. Which one of the following numbers is divisible by 9?

   (a) 7532458 (b) 6812348 (c) 6234588 (d) 4701828

6. 555555 is divisible by

   (a) 7 (b) 17 (c) 19 (d) 23

7. The prime factors of 15015 are

   (a) 3,5,7,11,37 (b) 3,5,11,13,37 (c) 3,5,7,13,37 (d) 3,5,7,11,13

8. If H.C.F. of $\left(x^2-4\right)$ and $\left(x^3-8\right) is \left(x-2\right)$ then their L.C.M. will be :

   (a) x^2+2x+4 (b) $\left(x^2-4\right)\left(x^3-8\right)$

   (c) $\left(x-2\right)\left(x^2-4\right)$ (d) $\left(x^2-4\right)\left(x^2+2x+4\right)$

9. The solution of the equation

   $x+y=3$

   $3x-4y=2$ are given by

   (a) $x=1, y=2$ (b) $x=3, y=0$ (c) $x=0, y=3$ (d) $x=2, y=1$

10. The region for which $x \geq 4$ is a part of the

    (a) first and second quadrants (b) second and third quadrants

    (c) third and fourth quadrants (d) fourth and first quadrants

11. The solution set of $x \leq 3, y \geq 0,$ and $x \leq -3, y \leq 0$ is

    (a) $x \geq -3, y=0$ (b) $x \leq -3, y=0$ (c) $x \geq -3, y \leq 0$ (d) $x \leq 3, y \geq 0$

12. Which one of the following statements is correct in respect of empty set ϕ ?

(a) $\phi = 0$ (b) $\phi = (0)$ (c) $\phi = \{\phi\}$ (d) $\phi = \{\ \}$

13. For any two sets A and B, the set $A \cap B$ is

(a) $\{x \mid x \in A \text{ or } x \in B\}$ (b) $\{x \mid x \in A \text{ or } x \notin B\}$

(c) $\{x \mid x \in A \text{ and } x \in B\}$ (d) $\{x \mid x \notin A \text{ and } x \in B\}$

14. $A \cap (B - A)$ is equal to

(a) A (b) B (c) ϕ (d) B – A

15. The number of diagonals of a hexagon is

(a) 6 (b) 8 (c) 9 (d) 10

16. In an angle equals two-third of its supplement, then the measurement of the angle is

(a) 36^o (b) 72^o (c) 108^o (d) 120^o

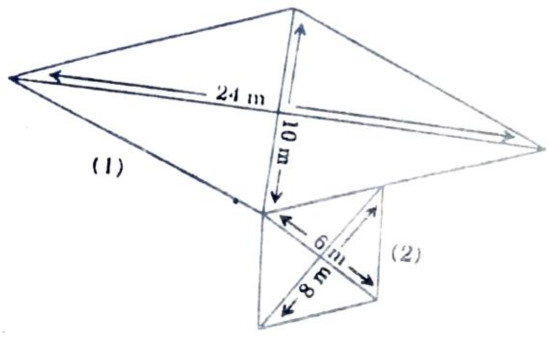
17. If the perimeter of a square is 16 cm then the area of the square is

(a) 8 sq.cm (b) 16 sq.cm (c) 32 sq.cm (d) 64 sq.cm

18. The sides of a rectangular field are in the ratio 3:4 and its area 7500 sq.m. The cost of fencing it at 25 paise per meter is

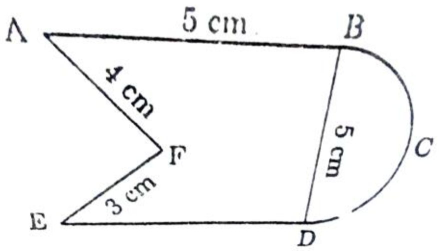
(a) ₹87.50 (b) ₹77.50 (c) ₹67.50 (d) ₹57.50

19. A field consists of two adjoining rhombus pieces. One has its diagonals 8 meters and 6 meters respectively while the second has its diagonals 24 meters and 10 meters respectively. The area of the field is



(a) 144 sq.m (b) 194 sq.m (c) 288 sq.m (d) 168 sq.m

20. The area of the given figure ABCDEF is



(a) 22.82 cm^2 (b) 25.82 cm^2 (c) 26.82 cm^2 (d) 28.82 cm^2

21. A's family income is 1440. If a pie chart is drawn to show how he spends under various heads such as food, clothing, education etc., then the amount spent on education, if it is represented by a sector of cntral angle 45^o, is

(a) ₹32 (b) ₹64.80 (c) ₹180

(d) cannot be determined, insufficient data.

22. In 1989 the number of workers in the Union were 4450 of which 4000 were male. The Non-Union workers were 800 of which 350 were female. The total number of male workers would be

(a) 400 (b) 800 (c) 5250 (d) 4450

23. The mean score of 12 students, in an examination, was 52.5. Later, on scrutiny, the scores of two students were changed from 45 and 51 to 40 and 62 respectively. The corrected mean score of all students is :

(a) 52.6 (b) 52.8 (c) 53.0 (d)53.5

24. The least common multiple of

$p(x) = (x-2)^2(x-2)$ and $q(x) = x^2 - 4x - 12$ is

(a) $(x+2)(x-2)$ (b) $(x+2)(x-2)(x-6)$

(c) $(x+2)(x-2)^2$ (d) $(x+2)^2(x-2)(x-6)$

25. The system of equations

$3x + y - 1 = 0$

$6x + 2y - 2 = 0$

(a) has x = 1 and y = 2 as a solution (b) has x = -1 and y = -2 as a solution

26. The ages of two persons differ by 20 years. 5 years ago, the elder one was 5 times as old as the younger one. Their ages are

(a) 30 years, 10 years (b) 25 years, 5 years

(c) 29 years, 9 years (d) 50 years, 30 years

27. A number consists of two digits whose sum is 15. If 9 added to the number, then the digits change their places. The number is

(a) 69 (b) 78 (c) 87 (d) 96

28. The values of x and y that simultaneously satisfy the equations $2x + 3y = 5$ and $7x - 4y = 3$ are

(a) 0,1 (b) 1,0 (c) -1,1 (d) 1,1

29. Consider the following statements :

1. Any set A is comparable with itself   2. {O} is a singleton set

3. (Ψ)is an empty set

Of these statement

(a) 1 and 2 are correct   (b) 1 and 3 are correct

(c) 2 and 3 are correct   (d) 1, 2 and 3 are correct

30. An angle which is greater than 180 but less than 360 is called a/an

(a) reflex angle   (b) right angle   (c) obtuse angle   (d) acute angle

31. Which one of the following statements is false?

(a) A line segment can be produced to any desired length

(b) Through a given point only one straight line can be drawn

(c) Through two given points it is possible to draw one, and only, straight line

(d) Two straight lines can intersect in only one point

32. Each angle of a regular pentagon will be

(a) 72^o (b) 90^o (c) 108^o (d) 120^o

33. Each internal angle of a regular hexagon is

(a) 60^o (b) 120^o (c) 108^o (d) 90^o

34. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is its

(a) incentre   (b) circumcenter   (c) orthocenter   (d) centroid

35. If the diagonals of a rhombus are 18 cm and 24 cm, then its side (in cm) is

(a) 9   (b) 12   (c) 15   (d) 18

36. Which one of the following statements is correct for a square?

(a) Diagonals are equal and bisect each other at right angles.

(b) Diagonals are unequal and do not bisect each other

(c) Diagonals are unequal and do not bisect each other

(d) Diagonals are unequal and bisect each other at right angles

37. Consider a point which moves such that its distances from two given points A and B are equal. Then the locus of the point P is

(a) a circle with centre at A   (b) a circle with centre at B

(c) a straight line passing through either A or B

(d)a straight line which is the right bisector of AB

38. Which one of the following is irrational?

(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{4}{5}$ (c) $\sqrt{7}$ (d) $\sqrt{81}$

39. Pipe A can fill a tank in 4 hours and pipe B can empty it icn 6 hours. If both of them are opened at the same time when the tank is empty, then number of hours required to fill the tank will be

(a) 6 (b) 8 (c) 10 (d) 12

40. The L.C.M. of $\frac{4}{5}, \frac{3}{10}$ and $\frac{7}{15}$ is

(a) $16\frac{4}{5}$ (b) $8\frac{2}{3}$ (c) $\frac{7}{15}$ (d) None of these above

41. The prime factors of 2310 are

(a) 2,3,4,7,11 (b) 2,4,5,7,11 (c) 2,3,5,7,11 (d) None of these

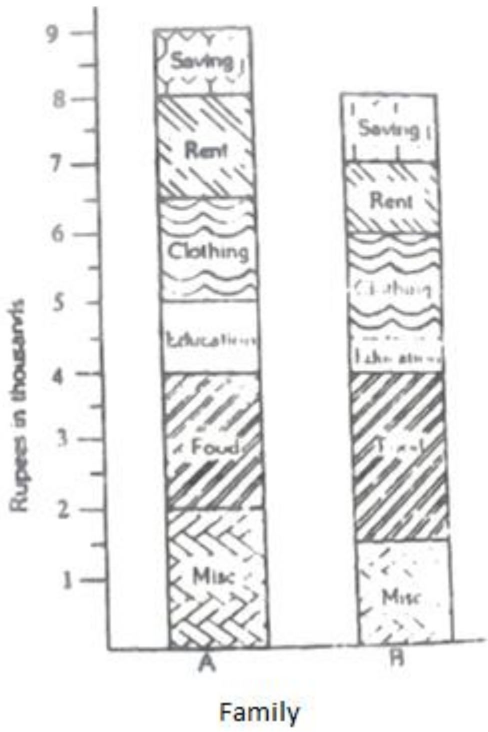
42. The age of a set-of people is tabulated in the form of a frequency distribution. Which one of the following increases with the increase in the width of the class interval?

(a) Total number of classes (b) Total frequency

(c) Accuracy of computed statistics (d) Frequency within a class

43. The given bar chart shows the monthly expenditure and savings of two families A and B. The difference in food expenditure of the two families is

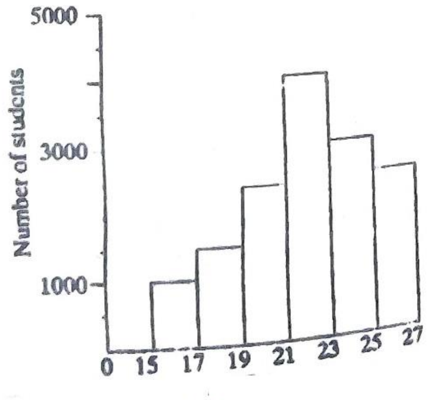
(a) ₹1000 (b) 800 (c) ₹750 (d) ₹500



44. The given histogram shows the strength of student age wise in a degree college. The

strength of the students between the age 21 to 23 years is

(a) 3,500 (b) 4,000 (c) 4,200 (d) 4,500



45. The mean deviation of the following distribution is

x	10	11	12	13	14
f	3	12	18	12	3

(a) 12 (b) 0.75 (c) 1.25 (d) 26

46. The mode of the given distribution is

Weight (in kg.) :	40	43	46	49	52	55
No. of children :	5	8	16	9	7	3

(a) 40 (b) 46 (c) 55 (d) None of the above

47. The standard deviation for the data 7, 9, 11, 13, 15 is

(a) 2.4 (b) 2.5 (c) 2.7 (d) 2.8

48. If $2x+3y=12$ and $3x-2y=5,$ then

(a) $x=9, y=-2$ (b) $x=3, y=2$ (c) $x=0, y=4$ (d) $x=-3, y=6$

49. The solution of the system of simultaneous linear equations $4x-3y=7; 5y+7x=2$ is

(a) $x=1, y-1$ (b) $x=-1, y=1$ (c) $x=-1, y=-1$ (d) $x=1, y=-1$

50. If A = [1,2,3,4] and B = [5,6,7], then $A \cap B$ is

(a) [1,2,3] (b) {5,6,7} (c) {4} (d) ϕ

51. Among the following statements, the incorrect one is

(a) if $A \subset B,$ then $A \cup B = B$ (b) if $A \subset B,$ then $A \cap B = A$

(c) $\{a,a,a\} = \{a\}$ (d) $A \cap (B \cup C) = A \cup (B \cap C)$

52. If in a class consisting of 100 students, 20 know English, 20 do not know Hindi, and 10

know neither English nor Hindi, then the number of students knowing both English and Hindi is

(a) 5 (b) 10 (c) 15 (d) 20

53. If P and Q are subsets of universal set X, then $(P \cup Q)'$ equals

(a) $P' \cap Q'$ (b) $P' \cup Q'$ (c) $P \cup Q'$ (d) $P' \cap Q$

54. If A, B, C are any three sets, then $A \cup B(B \cap C)$ is the same as

(a) $(A \cap C) \cup (B \cap C)$ (b) $(A \cup B) \cap (A \cup C)$

(c) $(A \cup B) \cap (B \cup C)$ (d) $(A \cap C) \cap (B \cup C)$

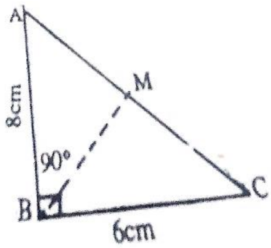
55. If A, B and C are any three non-empty sets, then $A - [B \cup C]$ equals

(a) $(A - B) \cup (A - C)$ (b) $(A - B) \cap (A - C)$

(c) $(A - C) \cup C$ (d) $(A - C) \cup B$

56. In the adjoining figure BM is a median of Δ ABC. It length (in cm) is :

(a) 3 (b) 4 (c) 5 (d) 7



57. In a right angled triangle, if the square of the hypotenuse is twice the product of the other two sides, then one of the angles of the triangle is

(a) 15^o (b) 30^o (c) 45^o (d) 60^o

58. PQR is an equilateral triangle, if *p, q* and *r* denote the lengths of the perpendicular from P, Q, R respectively, on the opposite sides, then

(a) $p \neq q \neq r$ (b) $p = q = r$ (c) $p \neq q = r$ (d) $p = q \neq r$

59. If ABCD is a parallelogram and E, F are the centroids of triangles ABD and BCD respectively, then EF equals

(a) AE (b) BE (c) CE (d) DE

60. If a square and a rhombus stand on the same base, then, the ratio of the areas of the square and the rhombus is

(a) greater than 1 (b) equal to 1 (c) equal to $\frac{1}{2}$ (d) equal to $\frac{1}{4}$

ANSWERS

1. (a)	2. (b)	3. (c)	4. (a)	5. (c)
6. (a)	7. (d)	8. (d)	9. (d)	10. (d)
11. (b)	12. (d)	13. (c)	14. (c)	15. (c)
16. (a)	17. (b)	18. (a)	19. (a)	20. (a)
21. (c)	22. (d)	23. (c)	24. (d)	25. (d)
26. (a)	27. (b)	28. (d)	29. (a)	30. (a)
31. (b)	32. (c)	33. (b)	34. (a)	35. (c)
36. (a)	37. (d)	38. (c)	39. (d)	40. (a)
41. (c)	42. (b)	43. (d)	44. (b)	45. (b)
46. (b)	47. (d)	48. (b)	49. (d)	50. (d)
51. (d)	52. (d)	53. (a)	54. (b)	55. (a)
56. (c)	57. (c)	58. (b)	59. (a)	60. (b)

SOLVED QUESTIONS –

(1) If a boy goes to school at 6 km/hr and returns home at 4 km/hr, find his average speed.

(A) Average speed = $\frac{2\times6\times4}{6+4}=\frac{48}{10}$ = 4.8 km/hr

(2) A man starts from B to K, another from K to B at the same time. After passing each other they complete their journeys in $3\frac{1}{3}$ and $4\frac{4}{5}$ hours, respectively. Find the speed of the second man if the speed of the first is 12 km/hr.

(A) $$\frac{1st\text{ man's speed}}{2nd\text{ man's speed}}=\frac{\sqrt{b}}{\sqrt{a}}=\sqrt{\frac{b}{a}}=\sqrt{\frac{4\frac{4}{5}}{3\frac{1}{3}}}=\sqrt{\frac{24}{5}\times\frac{3}{10}}=\sqrt{\frac{36}{25}}=\frac{6}{5}$$

$\therefore\ \frac{12}{\text{2nd man's speed}}=\frac{6}{5}$ $\therefore$ 2nd man's speed = $\frac{60}{6}=10$ km/hr.

(3) I shall be 40 min late to reach my office if I walk from my house at 3 km/hr. I shall be 30 min early if I walk at 4 km/hr. Find the distance between my house and the office.

(A) Let the usual time taken be 't' hours and speed be x km/hr.

Distance = xt = 3 x $5+\frac{40}{60}=4\left(t-\frac{30}{60}\right)$

$\therefore$ 3*t* + 2 = 4t - 2

$\therefore$ 4 = t

$\therefore$ Distance = $3\left(4+\frac{2}{3}\right)=14$ km.

(4) A man travels 120 km by ship, 450 km by rail and 60 km by horse taking altogether 13 hrs 30 min. The speed of the train is 3 times that of the horse and 1.1/2 times that of the ship. Find the speed of the train.

(A) If the speed of the horse is x km/hr; that of the train is 3x and that of the ship is

$\frac{3x}{1\frac{1}{2}}=2$ x km/hr

$\therefore\ \frac{120}{2x}+\frac{450}{3x}+\frac{60}{x}=\frac{27}{2}$ $\therefore\ \frac{60}{x}+\frac{150}{x}+\frac{60}{x}=\frac{27}{2}$

$\therefore\ \frac{270}{x}=\frac{27}{2}$ $\therefore$ x = 20 $\therefore$ Speed of the train = 60 km/hr

(5) A and B walk from P to Q, a distance of 21 km at 3 and 4 km/hr. B reaches Q, and

immediately returns and meets A at R. Find the distance from P to R.

(A) When they meet, both together have walked 2 x 21 = 42 km Since their speeds are as 3: 4, distances travelled are also as 3 : 4

$\therefore$ Distance travelled by A - PR = $\frac{3}{7}$ x 42 = 18 km.

(6) A train after travelling 50 km from A meets with an accident and proceeds at $\frac{4}{5}$ th of the former speed and reaches B, 45 min late. Had the accident happened 20 km further on, it would have arrived 12 min sooner (than if the accident occurred at C). Find the original speed and the distance.

(A) Let the speed be x km/hr.

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                              20
|---------------------------|---------|----------------------|
A                           C         D                      B
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When the speed becomes $\frac{4}{5}$ th of the usual, time taken would become $\frac{5}{4}$ th the usual, i.e., $\frac{1}{4}$ th more of the usual time.

So, $\frac{1}{4}$ th of the usual time taken to travel CB = 45 mm.

$\therefore$ $\frac{1}{4}$ th of usual time taken to travel CD (i.e. 20 km) = 12 min

$\therefore$ Usual time to travel 20 km = 48 min.

$\therefore$ Usual speed = 20 x $\frac{60}{48}$ = 25 km/hr.

Usual time taken to travel CB = 45 x 4 = 3 hrs.

$\therefore$ Distance CB = 25 x 3 = 75 km.

$\therefore$ Total distance = 50 + 75 = 125 km.

(7) A man travelling from A to B at 3 mph, takes half an hour rest at B, and returns to A at 5 mph. Total time taken is 3 hrs 26 min. Find the distance from A to B. (m = miles)

(A) Total time taken for travelling = 3 hrs 26 min. - 30 min. = 2 hrs. 56 min.

$= 2\frac{56}{60} = \frac{176}{60}$ hrs.

$\therefore$ Distance from A to B

$= \frac{176}{60}\left(\frac{3+5}{3+5}\right) = \frac{176}{60} \times \frac{15}{8} = 5.5$ miles

(8) Walking $\frac{8}{7}$th of his original rate, a man reaches his office 3 minutes early. Find the usual time he takes to reach office.

(A) $\left(1-\frac{7}{8}\right)$ x Usual time = Change in time

$\therefore$ Usual time = 3 x 8 = 24 minutes.

(9) The ratio between the speed of Meena and Teena is 2 : 3. Meena takes 20 minutes more than Teena to walk from A to B. If Meena had walked at double her speed, find the time she would take to walk from A to B.

(A) Ratio of speed of Meena and Teena is 2 : 3.

$\therefore$ Ratio of time taken = 3:2

If Teena takes x minutes to walk from A to B, then Meena takes x + 20 minutes.

$\therefore \frac{x+20}{x}=\frac{3}{2}$ $\quad \therefore 2x+40=3x$ $\quad \therefore x=40$ minutes.

$\therefore$ Meena takes 60 minutes walking at her usual speed.

$\therefore$ At double the speed, she would take 30 minutes.

(10) I row from A to B against the current in 8 hrs. and from B to A in 2 hrs. If the speed of the river is 9 m/sec., what is the speed of the boat in still, water?

(A) Let the speed of the boat in still water be x

$$\frac{\text{Time taken for up - journey}}{\text{Time taken for down - journey}}=\frac{8}{2}=\frac{4}{1}$$

$$\therefore \frac{\text{Speed for up-journey}}{\text{Speed for down-journey}}=\frac{1}{4}(\text{inverse})$$

$\therefore \frac{x-9}{x+9}=\frac{1}{4}$ (x is the speed of the boat)

$\therefore$ 3x = 45 $\quad \therefore$ x = 15 m/sec.

(11) A train travelling at 25 km/hr, leaves Delhi at 9 a.m. and another leaves Delhi at 35 km/hr. at 2 p.m. in the same direction. How many kms from Delhi do they meet?

(A) The first train has a start of 5 x 25 - 125 km. Relative speed = 35 - 25 = 10 km/hr.

$\therefore$ Time taken to meet = $\frac{\text{Distance}}{\text{Relative speed}}=\frac{125}{10}$ = 12.5 hrs. from 2 p.m.

$\therefore$ Distance from Delhi = $12\frac{1}{2}\times 35=437\frac{1}{2}$ kms.

(12) A man rows 27 km. with the stream and 15 km. against the stream taking 4 hrs each

time. Find his speed in km/hr in still water and the speed in km/hr at which the stream flows.

(A) Speed with the stream = $\frac{27}{4} = 6\frac{3}{4}$ km/hr.

$\therefore$ Speed against the stream = $\frac{15}{4} = 3\frac{3}{4}$ km/hr.

$\therefore$ Speed of the man in still water = $\frac{1}{2}\left(6\frac{3}{4} + 3\frac{3}{4}\right) = 5\frac{1}{4}$ = km/hr.

$\therefore$ Speed of the stream = $\frac{1}{2}\left(6\frac{3}{4} - 3\frac{3}{4}\right)$ = 1.5 km/hr.

(13) Two trains 121 metres long and 99 metres long are running in opposite directions, the first at 40 km/hr. and the second at 32 km/hr. In what time will they completely clear each other from the moment they meet?

(A) Total distance to be travelled = 121 + 99 = 220 metres.

Relative speed = Sum of speeds = 72 km/hr. = 72 x $\frac{5}{18}$ = 20 m/s.

$\therefore$ Time required = $\frac{220}{20}$ = 11 seconds.

(14) How long does a train 110 metres long running at 36 km/hr. take to cross a bridge 132 metres in length?

(A) Distance to be covered = 110 + 132 - 242 metres

Speed = $36 \times \frac{5}{18}$ = 10 m/sec. $\therefore$ Time taken = $\frac{242}{10}$ = 24.2 seconds.

(15) A car which was driven in fog passed a man walking at 3 km/hr. in the same direction. He could see the car for 4 minutes and upto a distance of 100 m. What was the speed of the car?

(A) Distance travelled by the man in 4 mins. = $\frac{4}{60} \times 3000 = \frac{12000}{60}$ = 200 m.

Distance travelled by the car in 4 mins. = 200 + 100 - 300 m.

$\therefore$ Speed of the car = $\frac{300}{4}$ m/minute = $\frac{300}{4 \times 1000} \times 60 = 4\frac{1}{2}$ km/hr.

(16) A person can row $7\frac{1}{2}$ km/hr. in still water. It takes him twice as long to row up a distance as to row down the same distance. Find the speed of the stream.

(A) Speed up-stream + Speed down-stream = 2 x $7\frac{1}{2}$ = 15 km/hr.

Since the times taken are in the ratio 2 : 1, the speeds will be in the ratio 1 : 2.

$\therefore$ Speed up-stream = $\frac{1}{3}$ x 15 = 5 km/hr.

Speed down-stream = $\frac{2}{3}$ x 15 = 10 km/hr.

Speed of stream = $\frac{1}{2}$ (10 – 5) = 2.5 km/hr.

Alternatively,

If the speed of the stream is x km/hr. = $\dfrac{7\frac{1}{2}+x}{2} = \dfrac{7\frac{1}{2}-x}{1}$

$\therefore$ x = 2.5 km/hr.

(17) A hare sees a dog 100 metres away from her, and scuds off in the opposite direction at a speed of 12 km/hr. A minute later, the dog sees the hare and chases the hare at a speed of 16 km/ hr. After how much time does the dog catch up with the hare?

(A) 12 km/hr. = 12 x $\frac{1000}{60}$ = 200 metres/min.

Distance of the hare from the dog when the dog sees the hare

= (100 + 1 x 200) = 300 metres

Since both are running in the same direction,

Relative speed (16 - 12) = 4 km/hr. = 4 x $\frac{1000}{60}$ m/min. = $\frac{200}{3}$ m/min.

$\therefore$ Time required to overtake = $\dfrac{300}{\frac{200}{3}} = \dfrac{300\times 3}{200} = 4\frac{1}{2}$ mins.

(18) A train leaving L at 3:10 p.m. reaches W at 5:00 p.m. One leaving W at 3:30 p.m. arrives in L at 5:50 p.m. At what time do they pass each other?

(A) Let the distance be d and let them meet t mins after 3:10 p.m. or (t - 20) mins, after 3:30p.m.

3:10 p.m. → d km. 5:00 p.m.

|--|

L t W

5:50 p.m. ← 3:30 p.m.

Their speeds are $\frac{d}{110}$ km/mins. and $\frac{3}{140}$ km/mins.

$\therefore$ distance = $\frac{d}{110} \times t + \frac{3}{140}(t-20) = d \qquad \therefore \frac{t}{110} + \frac{t-20}{140} = 1$

Solving t = 70.4 mins.

$\therefore$ They meet at 3:10 + 70.4 mins. = 4 hrs 20.4 mins.

(19) A train moving at uniform speed takes 20 secs, to pass a cyclist riding in the same direction at 11 km/hr but only 9 secs, to pass a post. Find the length of the train.

(A) If the length of the train is l km. and its speed x km/hr., then,

$$\frac{l}{x-11} = \frac{20}{3600} \qquad \text{... (i)}$$

and

$$\frac{l}{x} = \frac{9}{3600} \qquad \text{...(ii)}$$

Dividing (i) by (ii), $\frac{l}{x-11} \times \frac{x}{l} = \frac{20}{9} \therefore \frac{x}{x-11} = \frac{20}{9}$

$\therefore$ x = 20 km/hr.

$\therefore$ Length = 0.05 km = 50 metres.

(20) Which of the two numbers $(1.000001)^{1000000}$ and 2 is greater?

(A) $(1.000001)^{1000000} = \left(1 + \frac{1}{1000000}\right)^{1000000}$ which is greater than 2.

(21) Which of the two numbers 1000^{1000} and 1001^{999} is greater?

(A) $\frac{1001^{999}}{1000^{1000}} = \left(\frac{1001}{1000}\right)^{1000} . \frac{1}{1001} = \left(1 + \frac{1}{1000}\right)^{1000} . \frac{1}{1001}$

$$2 \le \left(1 + \frac{1}{1000}\right)^{1000} \le 3$$

$\therefore \left(1 + \frac{1}{1000}\right)^{1000} \times \frac{1}{1001} < 1 \therefore \frac{1001^{999}}{1000^{1000}} < 1 \therefore 1000^{1000} > 1001^{999}$

(22) Solve (-2 x + 3) $\le$ 6

(A) $-2x + 3 \le 6 \ \therefore -2x \le 3 \ \ \therefore 2x \ge -3 \ \ \therefore x \ge -\frac{3}{2}$

(23) If w satisfies both the following inequalities and w is an integer, what values can we have?

(i) 5 (w + 10) – 4w > 0 (ii) 8 + 7w < 3 (2w + 1)

(A) From (i) : 5 (w + 10) – 4w > 0

w + 50 > 0 $\therefore$ w > -50

From (ii) : 8 + 7w < 6w + 3

w < -5

$\therefore$ From (i) and (ii), w lies between -50 and -5 i.e. -50 < w < -5.

(24) Between what values of x, is the expression $19x – 2x^2 – 35$ positive?

(A) Let y denote the given expression

$$y = -\left(2x^2 - 19x + 35\right) = -(2x-5)(x-7)$$

$$= (2x-5)(7-x) = 2\left(x-\frac{5}{2}\right)(7-x)$$

(Refer property of quadratic inequalities)

For y to be positive

$$\left(x-\frac{5}{2}\right)(x-7) < 0 \therefore \frac{5}{2} < x < 7$$

(25) Find the range of value of x if $x^3 – 7x^2 + 16x – 10$ is positive.

(A) $x^3 - 7x^2 + 16x - 10$ has a factor x-1. Using synthetic division.

1	1	-7	16	-10
		1	-6	10
	1	-6	10	0

The second factor is $x^2 - 6x + 10$

$$\therefore\ x^3 - 7x^2 + 16x - 10 = (x-1)\left(x^2 - 6x + 10\right) = (x-1)\left[(x-3)^2 + 1\right]$$

$\left[(x-3)^2 + 1\right]$ is always positive.

$\therefore\ x^3 - 7x^2 + 16x - 10$ will be positive if x > 1.

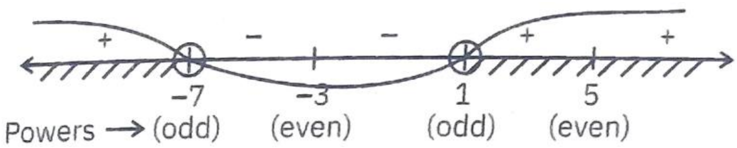
(26) Solve for $x : (x+7)^7 (x-5)^4 (x+3)^2 (x-1)^3 > 0$

(A) Equate LHS = 0

If $(x+7)^7 (x-5)^4 (x+3)^2 (x-1)^3 = 0$

$\therefore\ x = -7, -3, 1, 3.$

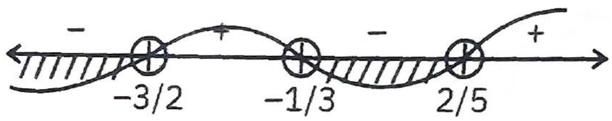
Draw the number line



The solution is $\left(-\infty,-7\right)\cup\left(1,\infty\right)$

(27) $\left(3x+1\right)\left(3+2x\right)\left(5x-2\right)<0$

(A) Equate LHS to zero. Then $x=-\frac{3}{2},-\frac{1}{3},\frac{2}{5}$



The solution is $x\in\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{1}{3},\frac{2}{5}\right)$

(28) $\left(3-2x\right)\left(1-5x\right)\left(-2-x\right)>0$

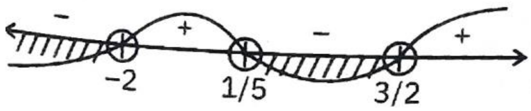
(A) $\left(3-2x\right)\left(1-5x\right)\left(-2-x\right)>0$

$\therefore\ \left(1\right)\left(2x-3\right)\left(-1\right)\left(5x-1\right)\left(-1\right)\left(x+2\right)>0$

$\therefore\ \left(-1\right)\left(-1\right)\left(-1\right)\left[\left(2x-3\right)\left(5x=1\right)\left(x+2\right)\right]>0$

$\therefore\ \left(2x-3\right)\left(5x-1\right)\left(x+2\right)<0$

Equate LHS to zero. Then $x=-2,\frac{1}{5},\frac{3}{2}$



The solution is $x\in\left(-\infty,-2\right)\cup\left(\frac{1}{5},\frac{3}{2}\right)$

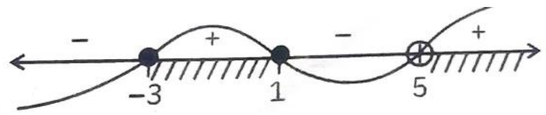
(29) $\frac{\left(x-1\right)\left(x+3\right)}{\left(x-5\right)}\geq 0$

(A) $\frac{\left(x-1\right)\left(x+3\right)}{\left(x-5\right)}\geq 0$

$\frac{\left(x-1\right)\left(x+3\right)\left(x-5\right)}{\left(x-5\right)^2}\geq 0$

Equate LHS to zero.

x = 1, -3, 5.



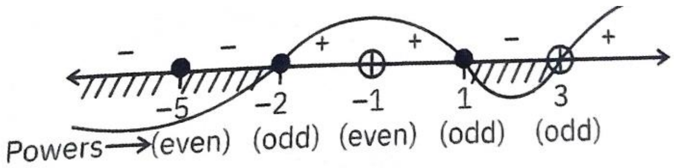
The solution is $x \in \left[-3,1\right] \cup \int \left(5,\infty\right)$

Note : As (x-5) is the denominator, x cannot be 5.

(30) $\dfrac{\left(x+2\right)^3\left(x-1\right)^5\left(x+5\right)^2}{\left(x+1\right)^4\left(x-3\right)} \le 0$

(A) The break points can be obtained directly by equating numerator and denominator of the LHS to 0.

$\therefore$ x = -2, 1, -5, -1, 3



The solution is

$x\left(-\infty,-2\right) \cup \left(1,3\right)$

Note : At x = 3 and x = -1 the graph will be discontinuous as the denominator is $(x+1)^4$ $(x-3)$. $\therefore$ x cannot take these values.

At x = -5, = -2 and x = 1, the graph cuts the x axis. Hence, the value of the expression on LHS is zero.

(31) $\left|x^2-5\right| < 7,$ then find the value of x

(A) $\therefore \left|x^2-5\right| < 7$

$\therefore -2 < x^2 < 12$

$-2 < x^2$ is always true.

As $x^2 < 12$

$x^2 - 12 < 0$

$\left(x+2\sqrt{3}\right)\left(x-2\sqrt{3}\right) < 0$

$-2\sqrt{3} < x < 2\sqrt{3}$

Hence the solution is $x \in \left(-2\sqrt{3}, 2\sqrt{3}\right)$

(32) If $\left|\dfrac{2x-1}{x+1}\right| < 3,$ then find the value of x.

(A) $\left|\frac{2x-1}{x+1}\right| < 3,$

$\therefore \left|\frac{2x-1}{x+1}\right| - 3 < 0$

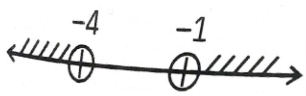
When $\frac{2x-1}{x+1} - 3 < 0$

$\therefore \frac{2x-1-3x-3}{x+1} < 0$

$\therefore \frac{-x+4}{x+1} > 0$

$\therefore \frac{x+4}{x+1} > 0$

$x < -4$ or x>-1



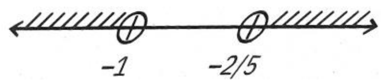
When - $\frac{2x-1}{x+1} - 3 < 0$

$\therefore \frac{-2x+1-3x-3}{x+1} < 0$

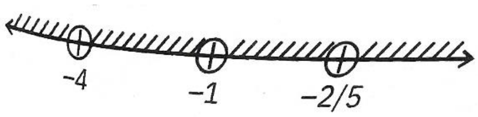
$\therefore \frac{-5x-2}{x+1} < 0$

$\therefore \frac{5x+2}{x+1} > 0$

$x < -1$ and $x > -2/5$



Combining the two cases we can conclude



x can be any real number except -1, -4 and $-\frac{2}{5}$.

(33) If $\left|\frac{x^2-9}{x+5}\right| > 0$, then find the value of x.

(A) The modulus value is always positive.

$\therefore \left|\frac{x^2-9}{x+5}\right|$ > 0 is true for all real values.

But x + 5 ≠ 0 i.e. x ≠ -5

Also $x^2 - 9 \neq 0$ i.e. x ≠ 3 & x ≠ -3

$\therefore$ The solution is $x \in R - \{-5, 3, -3\}$

(34) If $\frac{|x+3|+3}{x+2} > 1$, then find the value of x.

(A) $\frac{|x+3|+3}{x+2} > 1$

$\therefore \ \dfrac{|x+3|+x}{x+2} - 1 > 0$

$\therefore \ \dfrac{|x+3|-2}{x+2} > 0$

Case I :

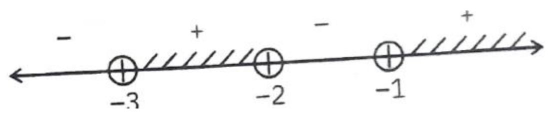
When x + 3 is positive,

$|x+3| = x+3 \ \ \& \ \ x > -3$

Again, $\dfrac{x+3-2}{x+2} > 0$

$\therefore \ \dfrac{x+1}{x+2} > 0$

The solution is



Hence, $x \in (-3,-2) \cup (-1,\infty)$

Case II :

When x + 3 is negative

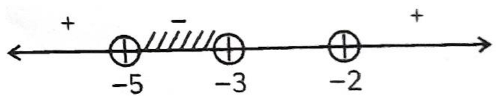
$|x+3| = 0-(x+3)$ and $x < -3$

Again, $\dfrac{=3(x+3)-2}{x+2} > 0$

$\therefore \ \dfrac{-x-5}{x+2} > 0$

$\therefore \ \dfrac{x+5}{x+2} < 0$

The solution is



Hence, $x \in (-5,-3)$

Case III :

When x + 3 = 0.

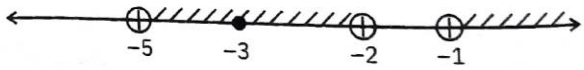
$|x+3| = 0 \qquad \therefore x = -3$

It gives $\dfrac{|-3+3|-3}{-3+2}$

$= \dfrac{0-3}{-1} = \dfrac{-3}{-1} = 3 > 0$

Hence, x = -3.

The final solution is



$x \in (-5,-2) \cup (-1,\infty)$

(35) Find the sum of 30 terms of the series 5 + 11 + 17 + ...

(A) The terms 5, 11, 17, ... form an Arithmetic Progression.

as d = 1 – 5 = 17 – 11 = 6 ; also, a = 5

$\therefore$ $S_{30} = \dfrac{n}{2}\left[2a+(n-1)d\right] = \dfrac{30}{2}\left[2\times5+29\times6\right]$

= 15 [10 + 174] = 2760.

(36) If $S_n = n(n+8)$, find T_1 and T_n

(A) $S_1 = T_1 = 1(1+8) = 9$

$S_{n-1} = (n-1)(n-1+8)$

$= (n-1)(n+7)$

$= n^2+6n-7$

$T_n = S_n - S_{n-1} = (n^2+8n)-(n^2+6n-7) = 2n+7$

(37) The sum of the first 31 terms of an A.P. is ‘0’. Which term of this A.P. will surely be a non-negative integer?

(A) In an A.P. with sum of all terms ‘0’, Sum of the first and last term, second and second-last term, third and third-last term and so on, will also be ‘0’. But whether these terms are positive or negative and whether they are integers or not, cannot be determined.

Since, the number of terms in the given A.P. is 31 (i.e. odd), the middle term (i.e. 16^{th} term) has to be ‘0’.

So, the 16^{th} term will surely be a non-negative integer.

(38) If the 4^{th} and the 9^{th} terms of a geometric progression are $\dfrac{1}{3}$ and 81 respectively, find the first term.

(A) $T_n = a\ r^{n-1}$

$T_4 = ar^3; \frac{1}{3} = ar^3$ ... (i)

$T_9 = ar^8; 81 = ar^8$ ... (ii)

Dividing (ii) by (i) ; 81 x 3 = r^5 $\therefore$ r = 3.

Substituting value of r in equation (i)

$\frac{1}{3} = a(3)^3 \qquad \therefore a = \frac{1}{81}$. Hence, the first term is $\frac{1}{81}$.

(39) How many terms of the series 1 + 5 + 9 + ... must be taken in order, so that the sum is 190?

(A) a = 1, d = 4.

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$190 = \frac{n}{2}\left[2 \times 1 + (n-1)4\right]$

380 = 2n + 4 n_2 - 4n

$2n^2$ - n – 190 = 0

(2n + 19) (n – 10) = 0

$\therefore$ n = $-\frac{19}{2}$ or n=10

n cannot be - $\frac{19}{2}$; $\quad \therefore n = 10$

(40) Find the geometrical progression whose sum to infinity is $\frac{9}{2}$ and the second term of which is – 2.

(A) $ar = -2 \ \therefore a = \frac{-2}{r}$

$\frac{a}{1-r} = \frac{9}{2}$

$\frac{-2}{r(1-r)} = \frac{9}{2}$

-4 = 9 r – $9r^2$

$9r^2 - 9r - 4 = 0$

$(3r+1)(3r-4) = 0$

$\therefore\ r = -\frac{1}{3}$ or $r = \frac{4}{3}$

If r = - $\frac{1}{3}$, a = - $\frac{2}{r} = 6$ and the series is 6, -2, $\frac{2}{3}$, ...

The value of r = $\frac{4}{3}$ is inadmissible, for r must be numerically less than unity.

(41) The number of bacteria in a culture triples in every 15 minutes. Find the number of bacteria in the culture after 105 minutes, if there were 10,000 bacteria initially.

(A) a = 10,000 ; r = 3 ; n = 8

$T_n = a\left(r^{n-1}\right) = 10,000\left(3^{8-1}\right)$

= 10,000 (2187) = 2187 x 10^4 bacteria.

(42) Find the value of K so that 8K + 3, 6K – 3 and 2K + 6 are in Arithmetic progression.

(A) If 8K + 3, 6K – 3 and 2K + 6 are in Arithmetic progression then (6K – 3) – (8K + 3)

= 2 (K + 6) – (6K – 3) ... [Common difference]

-2K – 6 – 4K + 9 $\quad\therefore$ K = $\frac{15}{2} = 7.5$

(43) Find the sum of the first 20 terms of the sequence 5, 5.5, 5.55, 5.555 ...

(A) $S_{20} = 5 + 5.5 + 5.55 + ... + T_{20}$

= $\frac{5}{9}\left[(10-1)+(10-0.1)+(1\ 0-0.01)+...+\left(10-10^{-19}\right)\right]$

= $\frac{5}{9}\left[10+10+10+... \text{ upto 20 terms}\right]$

- $\frac{5}{9}\left[1+\frac{1}{10}+\frac{1}{100}+... \text{ upto 20 terms}\right]$

= $\frac{5}{9}\left[10\times 20-1\left[\frac{1-\frac{1}{(10)^{20}}}{1-\frac{1}{10}}\right]\right]$

= $\frac{5}{9}\left[200-\frac{10}{9}\left[1-\frac{1}{10^{20}}\right]\right]$

= $\frac{5}{81}\left[1790+\frac{1}{10^{19}}\right] = \frac{5}{81}\times 1790...\left(\frac{1}{10^{19}}\approx 0\right) = 110.5$

(44) Find the sum of the first 10 terms of the sequence whose n^{th} term is 2n-8.

(A) $T_n = 2n-8$

$a = T_1 = 2 \times 1 - 8 = -6$

$T_2 = 2 \times 2 - 8 = -4$

$T_3 = 2 \times 3 - 8 = -2$

$-6, -4, 2$ form an A.P. $\therefore$ d = $T_2 - T_1$ = -4 – (-6) = 2

$$S_{10} = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{10}{2}\left[2 \times (-6) + 9 \times 2\right]$$

$$= 5\left[-12 + 18\right] = 30$$

(45) What will be the 27th term in the sequence : (3 x 5) + (5 x 8) + (7 x 11) + ...

(A) By observation, we can make out that each term in this sequence is multiplication of two expressions i.e. (2n+1) and (3n+2), where n = 1,2,3,...

Now, the nth term of the sequence T_n = (2n+1) x (3n+2)

$\therefore$ 27th term can be find out by putting n = 27.

T_{27} = (2 x 27 + 1) x (3 x 27 + 2) = 4565.

Note : In the above question, you will find that the second level difference is constant. So, you can also use the method explained earlier to derive the nth term of the sequence i.e. $T_n = an^2 + bn + c$).

SOLVED QUESTIONS -

1. In how many ways can 4 cards be selected from a pack of 52 cards so as to include at least one diamond card.

(A) Four cards can be selected from 52 cards in $^{52}C_4$ ways.

If none of these selected cards is a diamond card, they can be selected from the 39 cards in $^{39}C_4$ ways.

$\therefore$ The remaining selection will have at least one diamond card.

$\therefore$ Four cards with at least one diamond can be selected in $^{52}C_4$ - $^{39}C_4$.

Note: In a pack of cards there are 52 cards of four suits with 13 cards each. Two of the suits, diamond and hearts are red in colour and the other two, clubs and spades are black in colour. The 13 cards of each suit are valued as Ace, 2 to 10, lack, Queen and King.

2. A committee of 2 hawkers and 3 shopkeepers is to be formed from 7 hawkers and 10 shop- 'keepers. Find the number of ways in which this can be done if a particular shopkeeper is included and a particular hawker is excluded.

(A) A particular shopkeeper is included

$\therefore$ We have to choose 2 from remaining 9 in $^{9}C_{2}=\frac{9!}{7!2!}=\frac{8\times 9}{2}=36$ ways.

1 hawker is excluded

$\therefore$ We have to choose 2 hawkers from remaining 6 in $^{6}C_{2}=\frac{6!}{4!2!}=\frac{5\times 6}{2}$ = 15 ways

Total number of ways = 36 x 15 = 540.

3. In how many ways can a team of 11 cricketers be chosen from 6 bowlers, 4 wicket keepers and 11 batsmen to give a majority of batsmen if at least 4 bowlers are to be included and there is one wicket keeper?

(A) 1 wicket keeper from 4 can be selected in $^{4}C_{1}=\frac{4!}{3!1!}$ = 4 ways

If 4 bowlers are chosen then remaining 6 batsmen can be chosen in $^{11}C_6$.

$$^{6}C_{4}.^{11}C_{6}=\frac{6!}{4!2!}\times\frac{11!}{6!5!}=\frac{5\times 6}{2}\times\frac{11\times 10\times 9\times 8\times 7}{5\times 4\times 3\times 2}$$ = 15 x 14 x 33 = 6930

If we choose 5 bowlers then we have to choose 5 batsmen.

$\therefore$ There is no majority.

$\therefore$ Total number of ways = 4 x 6930 = 27720.

4. How many eight distinct letter words can be formed from the letters of the words "COURTESY" beginning with C and ending with Y?

(A) The first place will always be taken by C and last place always by Y. The remaining six places can be filled by the remaining six letters in $^6P_6$ ways.

$\therefore$ Total number of words - 1 x 1 x $^6P_6$ = 6! = 720

5. In how many ways can the seven letters A, B, C, D, E, F and G be arranged so that B and C are always together?

(A) Considering B and C as one, the six things can be arranged in 6P6 ways. B and C can be arranged among themselves in 2 ways, i.e., BC and CB.

Total number of arrangements = $^6P_6$ x 2 = 2 x 720 = 1440 ways.

6. How many 7 letter words can be constructed using the 26 letters of the alphabet series if each word contains exactly 3 vowels? (repetition is allowed)

(A) If there are 3 vowels, 3 places for 3 vowels can be chosen in $^7C_3$ ways, for each of the three places, there are 5 letters

$\therefore$ vowel positions = 5^3;

$\therefore$ remaining 4 places are occupied by 21 consonants

$\therefore$ consonant positions = 21^4.

$\therefore$ Number of words with 3 vowels is $^7C_3$ x 5^3 x 21^4

7. In how many ways can 7 Englishmen and 7 Americans sit around a circular table such that, no 2 Americans sit together?

(A) Putting 1 Englishman in a fixed position, the remaining 6 can be arranged in 6! = 720 ways. For each such arrangement, there are 7 positions for the 7 Americans and they can be arranged in 7! ways.

Total number of arrangements = 7! x 6! = 3628800

8. Find the number of ways of inviting at least one executive out of five executives to a conference.

(A) Here, we have two options for each executive - either he is invited or he is not invited So, the total number of ways of inviting these 5 executives = 2^5. But in these selections, there would be a case where all executives are not invited, which we need to exclude.

So, the total number of ways = 2^5 - 1 = 31

9. In how many ways can 21 identical white balls and 19 identical black balls be arranged in a row so that no 2 black balls are together?

(A) First arrange 21 white balls in a row. This can be done in 1 way (since they are identical). Now there are 22 places for the 19 black balls and so the places can be filled in

$^{22}C_{19}$ ways $= \frac{22!}{3!19!}$ ways OR $^{22}C_{3} = \frac{22\times21\times20}{2\times3} = 1540$

10. Find the probability that the birthday of a child is on a Saturday or Sunday.

(A) S = {Sun, Mon, Tue, Wed, Thur, Fri, Sat} $\therefore$ n(S) = 7

Let A be the event that the child is born on a Sunday or a Saturday.

$\therefore$ n(A) = 2

P(A) = $\frac{n(A)}{n(S)} = \frac{2}{7}$

11. Find the probability of getting a multiple of 2 in the throw of a die.

(A) S = {1, 2, 3, 4, 5, 6} $\therefore$ n(S) = 6

Let A be the event that the die shows a multiple of 2.

A = {2, 4, 6} $\therefore$ n(A) =

P(A) = $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

12. Find the probability that the sum of the scores obtained when two fair dice are thrown, is odd.

(A) S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4), (6, 5) (6, 6)}

$\therefore$ n(S) = 36

Let A be the event that the score on the two dice is odd.

$\therefore$ A = {(1, 2) (1, 4) (1, 6) (2, 1) (2, 3) (2, 5) (3, 2) (3, 4) (3, 6) (4, 1) (4, 3) (4, 5) (5, 2) (5, 4) (5, 6) (6, 1) (6, 3) (6, 5)}

$\therefore$ n(A) = 18

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

Note:

When a fair die is thrown, n(S) = 6, when two fair dice are thrown, n(S) = 6^2 = 36, when three fair dice are thrown, n(S) = 6^3 = 216.

13. A committee of 5 students is to be chosen from 6 boys and 4 girls. Find the probability

that the committee consists of exactly 2 girls.

(A) 5 students can be selected from 10 in $^{10}C_5$ ways.

$$\therefore\ n(S) = {}^{10}C_5 = \frac{10!}{5!5!} = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} = 252$$

Let A be the event that the committee includes exactly 2 girls and 3 boys. The two girls can be selected in $^4C_2$ ways and the 3 boys can be selected in $^6C_3$ ways.

$\therefore$ n(A) = $^4C_2$ x $^6C_3$ = 6 x 20 = 120

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{120}{252} = \frac{10}{21}$$

14. A bag contains 7 red, 5 blue, 4 white and 4 black balls. Find the probability that a ball drawn at random is red or white.

(A) A ball can be selected from 20 in $^{20}C_1$ = 20 ways.

$\therefore$ n(S) = 20

Let A be an event that the ball drawn is red or white.

$$\therefore\ n(A) = {}^7C_1 + {}^4C_1 = 7 + 4 = 11 \qquad \therefore P(A) = \frac{11}{20}$$

15. Two cards are drawn at random from a pack of 52 cards. What is the probability that one of them is a Jack card and the other is an Ace card?

(A) Two cards can be selected in $^{52}C_2$ ways.

$\therefore$ n(S) = $^{52}C_2$ = 26 x 51

Let A be the event that the two cards selected are a Jack and an Ace.

A Jack card can be selected in $^4C_1$ ways and an Ace card can also be selected in $^4C_1$ ways.

$\therefore$ n(A) = $^4C_1$ x $^4C_1$ = 16

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{16}{26\times51} = \frac{8}{663}$$

16. In a shooting competition, the probability that the target is hit by A is $\frac{2}{5}$, by B is $\frac{2}{3}$ and by C is $\frac{3}{5}$. If all of them fire independently at the same target, calculate the probability that only one of them will hit the target.

(A) P(A) = $\frac{2}{3}$ P(A') = $\frac{3}{5}$

P(B) = $\frac{2}{3}$ P(B') = $\frac{1}{3}$

P(C) = $\frac{3}{5}$ P(C') = $\frac{2}{5}$

Probability that only one of them hits the target.

= Probability that A hits the target but not B and C.

+ probability that B hits the target but not A and C.

+ probability that C hits the target but not A and B.

= $P\left(A\cap B'\cap C'\right)+P\left(A'\cap B\cap C'\right)+P\left(A'\cap B'\cap C\right)$

= $\left[\frac{2}{5}\times\frac{1}{3}\times\frac{2}{5}\right]+\left[\frac{2}{3}\times\frac{3}{5}\times\frac{2}{5}\right]+\left[\frac{3}{5}\times\frac{3}{5}\times\frac{1}{3}\right]=\frac{4}{75}+\frac{12}{75}+\frac{9}{75}=\frac{25}{75}=\frac{1}{3}$

17. A box contains 6 white balls and 3 black balls, and another box contains 4 white balls and 5 black balls. Find the probability that the ball randomly selected from one of the boxes, is a white ball?

(A) Let A be the event that the first box is selected, B be the event that the second box is selected and C be the event that a white ball is selected.

P(A) = $\frac{1}{2}$ and P(B) = $\frac{1}{2}$

P(C/A) = $\frac{6}{9}$; P(C/B) = $\frac{4}{9}$

The probability of selecting the first box and getting a white ball

= $P\left(A\right)=P\left(A.P\left(C/A\right)\right)=\left(\frac{1}{2}\right)\left(\frac{6}{9}\right)=\frac{6}{18}$

The probability of selecting the second box and getting a white ball

= $P\left(B\cap C\right)=P\left(B\right).P(B/C)=\left(\frac{1}{2}\right)\left(\frac{4}{9}\right)=\frac{4}{18}$

because The event $\left(A\cap C\right)$ and $\left(B\cap C\right)$ are mutually exclusive events.

$\therefore\ P\left[\left(A\cap C\right)\cup\left(B\cap C\right)\right]=\frac{6}{18}+\frac{4}{18}=\frac{10}{18}=\frac{5}{9}$

Alternatively,

Probability of selecting any bag is $\frac{1}{2}$.

Probability of getting a white ball will be = $\frac{1}{2}\times\frac{{}^{6}C_1}{{}^{9}C_1}+\frac{1}{2}\times\frac{{}^{4}C_1}{{}^{9}C_1}=\frac{5}{9}$

18. In a garden, 40% of the flowers are roses and the rest are carnations. If 25% of the roses and 10% of the carnations are red, find the probability that a red flower selected at random is a rose.

(A) Suppose there are 100 flowers.

Number of roses - 40 and number of carnations = 60

25% of 40 = 10 roses are red and 10% of 60 = 6 carnations are red.

Let A be the event that the flower is red and B be the event that the flower is a rose.

$\therefore \; A \cap B$ is the event that the flower is a red rose.

$n(A)16; \qquad \therefore P(A) = \frac{16}{100}$

$n(A \cap B) = 10; \qquad \therefore P(A \cap B) = \frac{10}{100}$

P(B/A) = probability that the selected flower is a rose given that the flower is red in colour.

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{10/100}{16/100} = \frac{10}{16} = \frac{5}{8}$$

19. Two sisters A and B appeared for an audition. The probability of selection of A is $\frac{1}{5}$ and that of B is $\frac{2}{7}$. Find the probability that both of them are selected.

(A) Let A be the event that A is selected and B be the event that B is selected.

$\therefore$ P(A) = $\frac{1}{5}$ and P(B) = $\frac{2}{7}$

Let C be the event that both are selected.

$\therefore \; C = A \cap B; \qquad \therefore P(C) = P(A \cap B)$

$\therefore$ P(C) = P(A). P(B) as A and B are independent events $= \frac{1}{5} \times \frac{2}{7} = \frac{2}{35}$

20. Nikita draws a ball randomly from a bag containing 4 white and 5 black balls. What are the Odds against it being a black ball?

(A) Total favourable outcomes (i.e. getting a black ball) = 5

Total Unfavourable outcomes (i.e. getting a ball other than black color) = 4

So, Odds against the ball drawn being black = $\frac{4}{5}$.

DIRECTIONS for questions 1 to 6: Refer to the data below and answer the questions that follow:

In a class, there are 30 boys and 20 girls. 20 of the boys and 5 of the girls drink coffee. Two students X and Y are picked at random.

1. What is the probability that X is a girl who does not drink coffee?
2. What is the probability that X drinks coffee, given that Y drinks coffee?
3. What is the probability that X drinks coffee, given that X is a girl?
4. What is the probability that X is a girl, given that X drinks coffee?
5. What is the probability that X and Y both drink coffee?
6. What is the probability that X and Y are a girl and a boy who both drink coffee?

ANSWERS-

1. (0.3) 2. (24/49) 3. (0.25) 4. (0.2)

5. (12/49) 6. (4/49)

DIRECTIONS for questions 7 to 21: Choose the correct alternative.

7. Find the chance of throwing more than 15 in one throw with 3 dice.

(1) $\frac{1}{54}$ (2) $\frac{17}{216}$ (3) $\frac{5}{108}$ (4) Cannot be determined

8. A wardrobe consists of 4 shirts, 5 T-shirts and 6 trousers. Of these items of clothing, 3 are selected. What is the probability that one from each is selected?

(1) $\frac{76}{91}$ (2) $\frac{3}{455}$ (3) $\frac{24}{91}$ (d) None of these

9. Jitu invites Sudhir for a party. Sudhir knows that on the way to Jitu's house he'll come across four lanes. There are five houses at the end of each lane. One of these five houses is Jitu's. Find the probability that the first house that Sudhir checks is not Jitu's.

(1) $\frac{1}{20}$ (b) $\frac{19}{20}$ (3) $\frac{4}{5}$ (4) $\frac{1}{5}$

10. In a group consisting of 12 boys, there are 6 boys two each of whom share the names Suresh, Mahesh and Jayesh. The rest of the six boys have six different names, other than the names mentioned. Four boys from the group are selected for a party such that the boys with the same names are always together (i.e., either all of them go or none of them goes). Find the probability that the boys named Suresh go for the party.

(1) $\frac{3}{41}$ (2) $\frac{10}{41}$ (3) $\frac{17}{63}$ (4) $\frac{15}{63}$

11. In a certain survey it was found that 20% of the women and 40% of the men smoke. Women constitute 45% of the population surveyed. If a person is selected at random and it was found that he or she smokes, determine the probability that the person

(1) $\frac{22}{31}$ (2) $\frac{2}{5}$ (3) $\frac{9}{22}$ (4) $\frac{9}{31}$

12. In a three section test, thee probability that Mr. X clears first section, second section and third section is $\frac{1}{5}$, $\frac{2}{3}$ and $\frac{3}{4}$ respectively. Find the probability that Mr. X clears exactly one section in the test.

(1) $\frac{7}{20}$ (2) $\frac{1}{4}$ (3) $\frac{7}{36}$ (4) $\frac{13}{95}$

13. There are two bags, one red and the other black. The red bag contains 9 balls, of which 4 are yellow. The black bag contains 17 balls, of which 9 are yellow. The probability of selecting a red bag is $\frac{4}{7}$ while the probability of selecting a black bag is $\frac{3}{7}$. Find the chance that when a ball is selected at random, it is not yellow.

(1) $\frac{152}{357}$ (2) $\frac{498}{1071}$ (3) $\frac{560}{1071}$ (4) $\frac{556}{1071}$

14. In a chess tournament, three matches are played between A and B. The probability that A wins a match is $\frac{1}{2}$ and the probability that A draws a match is $\frac{1}{3}$. Find the probability that A and B win alternately in the tournament such that there is no draw.

(1) $\frac{1}{18}$ (2) $\frac{1}{24}$ (3) $\frac{1}{36}$ (4) $\frac{1}{48}$

15. What is the probability of choosing a two-digit prime number such that the sum of its digits is odd?

(a) $\frac{3}{7}$ (2) $\frac{8}{21}$ (3) $\frac{10}{21}$ (4) $\frac{4}{7}$

16. The odds in favour of my clearing the 3 stages of admissions to MBA viz., CAT, GD and PI, in that order are 3 to 2, 2 to 3 and 4 to 1 respectively. What is the probability that I clear at least 2 of the 3 stages? (Given that selection to the next stage depends on clearing the previous one)

(1) $\frac{82}{125}$ (2) $\frac{27}{50}$ (3) $\frac{6}{25}$ (4) $\frac{36}{75}$

17. A and B are among the participants of a dart shooting competition. The probability that

A wins is $\frac{2}{9}$ and the probability that B does not hit the target in the 'bulls eye' is $\frac{1}{3}$. Assume that a 'win' is hitting the 'bulls eye'. The probability of dead heat (i.e. all of them win) is $\frac{1}{27}$ If there are 8 contestants in the competition and probability of all others (except A and B) winning is equal; then what is the probability that no one wins?

(1) $\frac{7}{108}\left(\sqrt[6]{4}-1\right)$ (2) $\frac{7}{108}\left(\sqrt[3]{2}-1\right)^6$ (3) $\frac{7}{27}\left(\sqrt[6]{4}-1\right)$ (4) $\frac{7}{27}\left(\sqrt[3]{2}-1\right)^6$

18. In a company, the total number of employees is 100. A "Grade A" employee can avail of any combination of a house loan, personal Loan and an automobile loan. A "Grade B" Employee can avail of any combination of a personal loan and an automobile loan. A "Grade C" employee can only avail of an automobile loan. There are no loan privileges for "Grade D" employees. Also, the probability of obtaining at least two loans by an employee selected at random is $\frac{1}{2}$ and the probability of obtaining at least one loan is $\frac{7}{10}$. What is the probability that an employee selected at random can obtain exactly one loan?

(1) 0.1 (2) 0.2 (3) 0.3 (4) 0.4

19. "Suppose all the teams batting first in one-day international cricket score between 200 and 399 runs (both included) and each score between 200 and 399 is equally likely, what is the probability that Team A scored the same number of runs in two matches when batting first?

(1) 1% (2) 0.5% (3) 2%

(4) More information is needed to answer this question

20. Two squares are chosen at random on a 8 x 8 chessboard. What is the probability that they are adjacent and touching each other diagonally?

(1) $\frac{43}{504}$ (2) $\frac{7}{144}$ (3) $\frac{7}{12}$ (4) $\frac{61}{504}$

21. An unbiased die is thrown thrice. What is the probability that the sum of the numbers appearing on it is a perfect square?

(1) $\frac{17}{108}$ (2) $\frac{4}{27}$ (3) $\frac{5}{36}$ (4) $\frac{11}{72}$

ANSWERS-

7. (3)	8. (3)	9. (2)	10. (3)	11. (1)
12. (1)	13. (4)	14. (1)	15. (1)	16. (3)
17. (2)	18. (2)	19. (2)	20. (2)	21. (1)